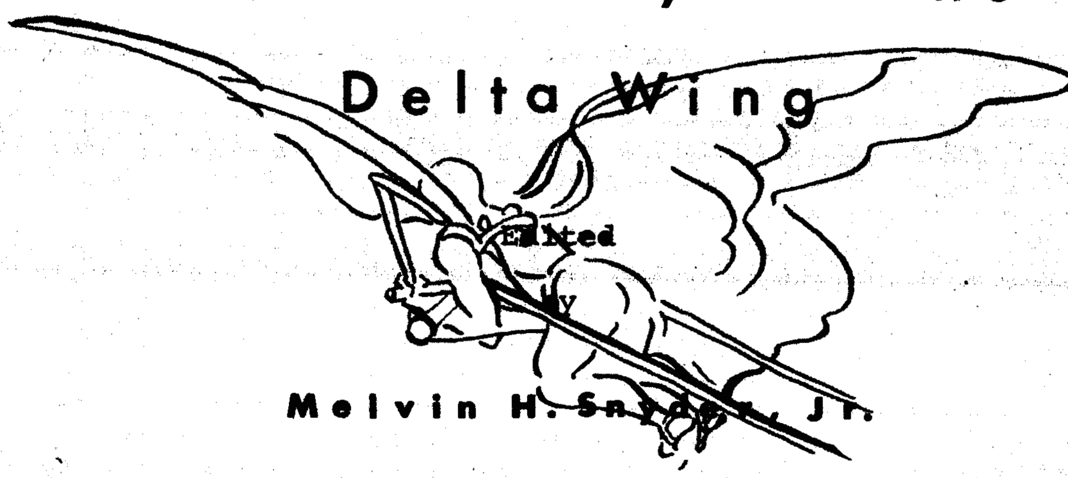
AR 66-4

On The Theory Of The Delta Wing



Edited

by

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Department of Aeronautical Engineering
WICHITA STATE UNIVERSITY

September, 1966

ON THE THEORY OF THE DELTA WING

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Aeronautical Report 66-4

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Department of Aeronautical Engineering
School of Engineering
Wichita State University
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ABSTRACT

Contributions to the theory of delta wings by three papers published by the O. N. E. R. A. are discussed. These papers are concerned with (1) the shape of the vortex sheets shed by the sharp leading-edges of thin delta wings, (2) the "bursting" of the vortices, i.e., the flaring of the vortex cores, and (3) the effect of these phenomena on the lift of delta wings.

Complete translations of two of the papers are presented:

M. Roy: "On the Theory of the Delta Wing—Apex-Vortices and Sheets _en Cornet_."

H. Werle: "On the Bursting of the Apex-Vortices of a Delta Wing at Low Speeds."

CONTENTS

treated by Poisson-Quinton and Erlich in "Hyper-sustentation et equilibrage des ailes élancees" -- presented at an O. N. E. R. A. Colloquium in November, 1964.

The paper by Poisson-Quinton has been translated as NASA TT F-9523 (reference 4). The papers by Roy and Werle are presented in translation in this report.

Maurice Roy is the Director-General of the O. N. E. R. A. His paper, "On the theory of the delta wing -- Apex-vortices and sheets *en cornet*", describes, in detail, the vortex sheets which separate from the sharp leading-edge of a thin delta wing. These vortex sheets roll-up into a helical - or cone-shaped vortex that is cornucopia-shaped and which apparently originates at the apex of the delta wing. These vortices, Mr. Roy terms "cornets" (after the French pastry cornets which are small funnel-shaped pastries, often filled with whipped cream). This term is so descriptive, that in the translation, included in this report, it is preserved and reference is made to vortex sheets *en cornet*.

He goes to considerable length in describing the flow field around a thin, flat delta wing with sharp leading-edges operating at a positive angle of attack. The analysis can be summarized as follows: There is produced, on both the upper and lower surfaces, a transverse flow. The pressure

INTRODUCTION

During the past two years, studies of delta wings have been carried on at Wichita State University under NASA grants. These studies included extensive experimental studies of the flow fields about delta and double-delta wings conducted at low speed and reported by Wentz in references 1 and 2. Also a review of the literature was made by Razak and Snyder (reference 3).

In the course of this work, it became evident that three important concepts in this field had been treated in papers published by L'OFFICE NATIONAL D'ETUDES ET DE RECHERCHES AERONAUTIQUES (O. N. E. R. A.) of France. These concepts are:

(1) The geometry of the vortex sheet which separates from the sharp leading-edge of a thin delta wing — treated by Maurice Roy in "Sur la theorie de l'aile in delta", published by O. N. E. R. A. in February, 1957.

(2) The "bursting" of the vortices above delta wings — treated by H. Werle in "Sur l'eclatement des tourbillons d'apex d'une aile delta aux faibles vitesses," published by the O. N. E. R. A. in 1960.

(3) Limitation to the value of lift-curve slope due to vortex bursting and a method for predicting lift —

field causes the fluid to flow around the sharp leading-edge from the lower surface. This sharp leading-edge constitutes a singular point in the transverse plane — a point which cannot exist in a real fluid flow field. The surface fluid (parietal fluid) separates from the surface at or near the leading-edge forming vortex filaments which constitute a vortex sheet. This vortex sheet wraps-up *en cornet*, providing an extension of the thin wing and terminating in a *bourrelet tourbillonnaire marginal de la nappe* — a vortex pad (or bubble) edge of the sheet. This phenomenon, which is also described as "the padded marginal edge of the sheet," is really a rotational core of the wrapped-up vortex sheet.

In the paper, Roy attempts to mathematically describe this actual flow field (figure I-a) with results that are not, at present, useful. In contrast, Brown and Michaels approximated it with a simplified pattern (figure I-b) before describing the field mathematically with quite useful results.

Although Roy refers to "...particular phenomena which appear near the tips... of these wings as well as in certain regions of their trailing-edge,...", he then ignores these phenomena and concerns himself only with the "... two symmetric vortices coming especially from the apex and forming, above the upper surface, a vee...". In so doing, Roy has ignored the secondary vortices. Figures II and III, taken from reference 3, show schematic representations of the secondary vortex.

Wentz measured the effect of the secondary vortex by integrating the circulation about paths which included and excluded this vortex. He reported, in reference 2, that the value of circulation determined by performing an integration along a closed path which excluded the secondary "reversed" vortex was larger than that obtained by choosing a path of integration which included the secondary vortex. The latter value is that corresponding to measured lift of the wing.

"Flow field measurements generally confirm the patterns assumed in theoretical models except for the presence of a secondary, reversed vortex. Vortex strength values and vortex spans are considerably less than those predicted by mathematical models. The reversed vortex does not appear to be of sufficient magnitude to account for the discrepancy between theory and experiment. The onset of vortex core breakdown and attendant reduced circulation seem to be responsible for the greatest discrepancy between experiment and theory." -(2)

H. Werle, in "On the bursting of the apex-vortices of a delta wing at low speeds", presents an excellent study of the "bursting" or "explosion" of the vortex sheet <u>en cornet</u> above the wing surface. This phenomenon is called "vortex breakdown" by many authors. The terms "bursting" and vortex "breakdown" are misleading in that they imply a more or less complete disorganization of the flow with immediate disappearance of circulation. This disintegration is not what occurs. Wentz has suggested that vortex-core flaring is a more accurate term to describe this phenomenon. The rotational core of the vortex increases in diameter; the circulation does not immediately dissappear, although the rate of viscous dissipation increases.

In "Hyperlift and Balancing of Slender Wings", Phillipe Poisson-Quinton and E. Erlich show the effect of the flaring of the vortex core on the wing lift. Reference 2 states:

"Results indicate that vortex core breakdown is the source of the principal discrepancy between measured and theoretical lift. The lift prediction method of Poisson-Quinton accounts for the breakdown and seems to be the most satisfactory method of lift prediction available presently. Better definition of vortex core breakdown boundaries for non-delta slender wings and the effects of breakdown on lift will be required before lifting prediction for slender sharp-edged wings is entirely satisfactory."

The translation of Roy's paper was by Professor-Emeritus Jacquetta Downing, and of the Werle paper by Miss Nancy Razak.

REFERENCES

1. Wentz, William H., Jr.; and McMahon, Michael C.: An Experimental Investigation of the Flow Fields about Delta and Double-Delta Wings at Low Speeds. AR 65-2, Aero. Engr. Dept., Wichita State University, August 1965.

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5. Brown, Clinton E.; and Michael, William H., Jr.: On Slender Delta Wings with Leading-Edge Separation. NACA TN 3430, 1955.

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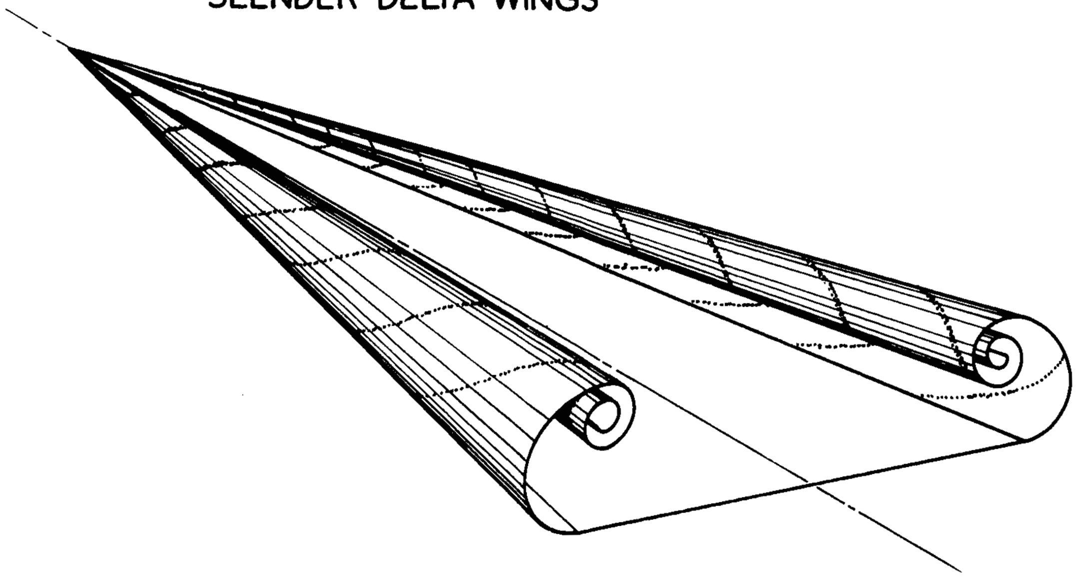
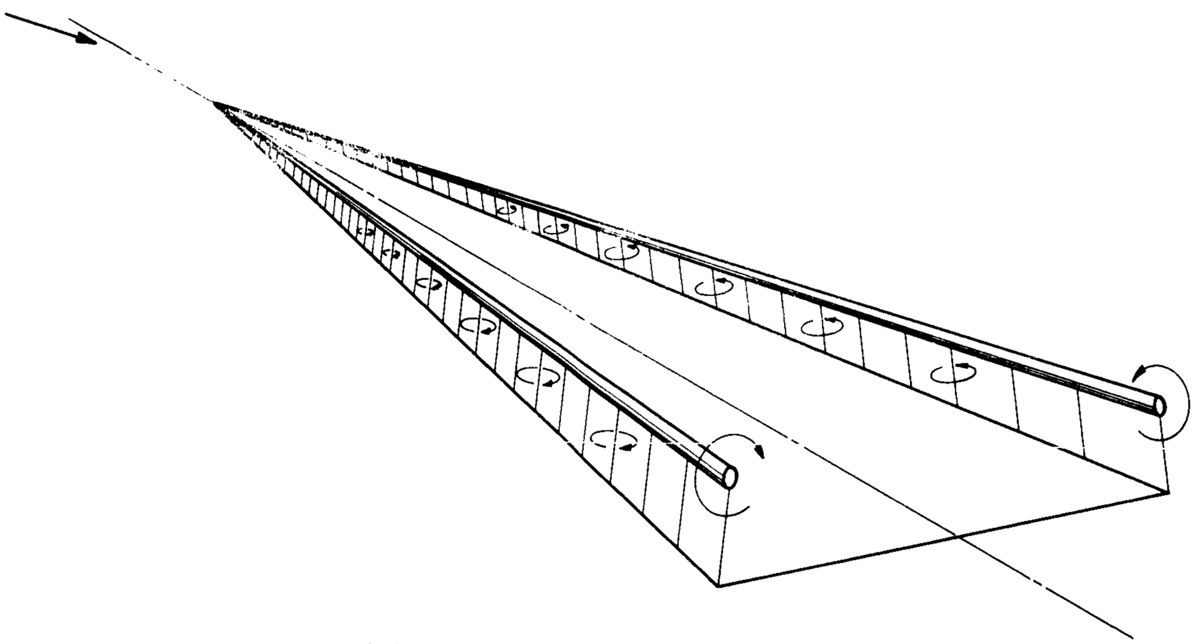
SCHEMATIC DRAWINGS OF SEPARATED FLOW OVER SLENDER DELTA WINGS



(a) ASSUMED FLOW FIELD



(b) APPROXIMATED FLOW FIELD

Figure I

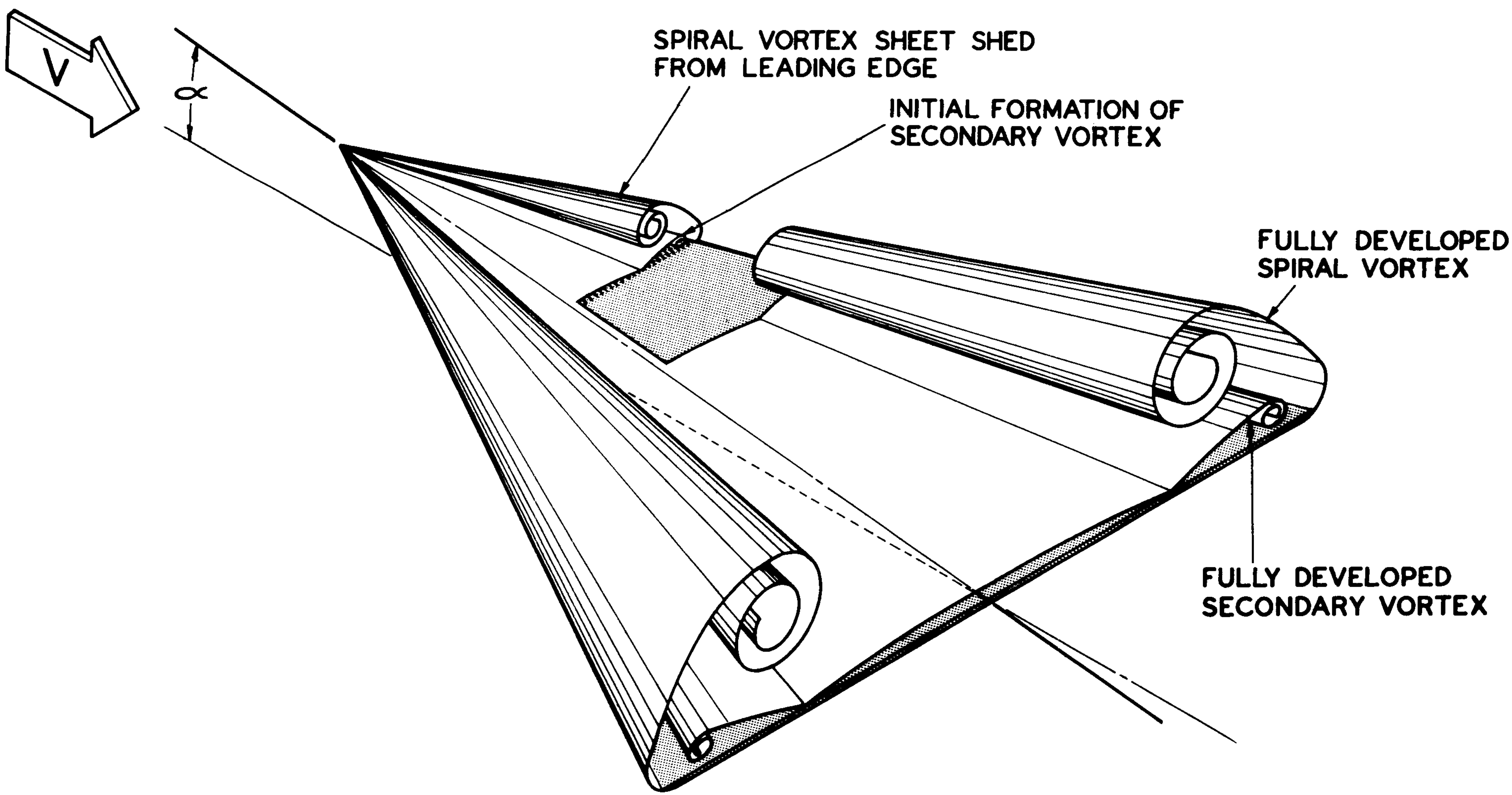

V
α
SPIRAL VORTEX SHEET SHED FROM LEADING EDGE
INITIAL FORMATION OF SECONDARY VORTEX
FULLY DEVELOPED SPIRAL VORTEX
FULLY DEVELOPED SECONDARY VORTEX

Figure II

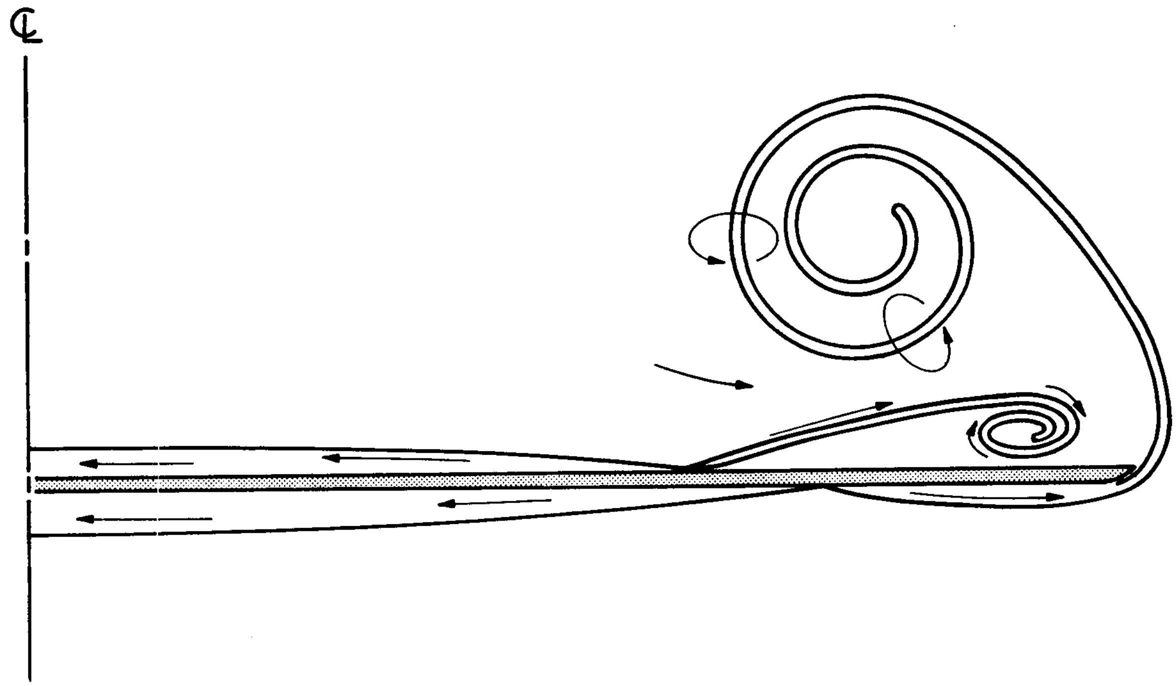


Figure III

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Translation of: SUR LA THEORIE DE'LAILE EN DELTA--TOURBILLONS D'APEX ET NAPPES EN CORNET

From: La Recherche Aeronautique, No. 56

Fevrier, 1957

ON THE THEORY OF THE DELTA WING

VORTICES FROM THE APEX AND SHEETS IN CORNET

by

Maurice Roy

Member of the (French) Academy of Sciences

Director-General of the O.N.E.R.A.

1. INTRODUCTION

For some time numerous works have been published in order to establish, for delta and other wings with extreme sweep-back, a simplified theory which may be in harmony with the peculiarities, more and more widely known, of the flow of air around wings of this sort.

It is essentially of the scheme of this flow that I shall treat here with a view of utilizing it as a basis of theory.

The observation of the phenomena must necessarily precede the elaboration of their theory. For at least six years I have given, through the O.N.E.R.A., much emphasis on research on the visualization of these flows. In this manner (and thanks notably to the efforts and to the cleverness of Mr. Werle who has emphasized, among others, the technique of opaque filaments or streaks, white or colored), the hydrodynamic tunnel at Chatillon, following my conception, was used intensely and resulted in the valuable cross-checking of other attempts. These other efforts were expended in wind tunnels and were accompanied by visualization through the use of smoke, volatile liquids and wool tufts.

In the hydrodynamic tunnel, especially, it was possible to examine thoroughly some details which otherwise escape observation. Mr. Werle has already presented in Recherche Aeronautique, at two different times, some results of various studies of this kind.

In passing, I emphasize that in this research there has been no ignoring of the considerable difference between the Reynolds' Numbers realized in these studies in the hydrodynamic tunnel and those relative to a real plane wing, even if the latter flies only at speeds at which the compressibility of the air is legitimately negligible.

But with the necessary changes, these experiments furnish on the development of some phenomena, qualitative information that one might qualify (with some humor) by saying that the colored streaks, are, in the experiments in question, intensely enlightened.

2. THE SCHEME OF THE POTENTIAL FLOW FIELD

The delta wing considered here is constituted by an angular sector of an infinite plane with the angle at the top being $\pi - 2\varphi$, φ being the angle of sweepback.

The relationship of the xyz axes to the wing are shown in Figure 1; the axis, x, is the longitudinal axis of the wing, positive in the downstream direction. The flow is assumed steady and the speed at infinity, $\overline{V}_o$ at the angle of incidence and with no drag. The fluid is assumed incompressible and perfect.

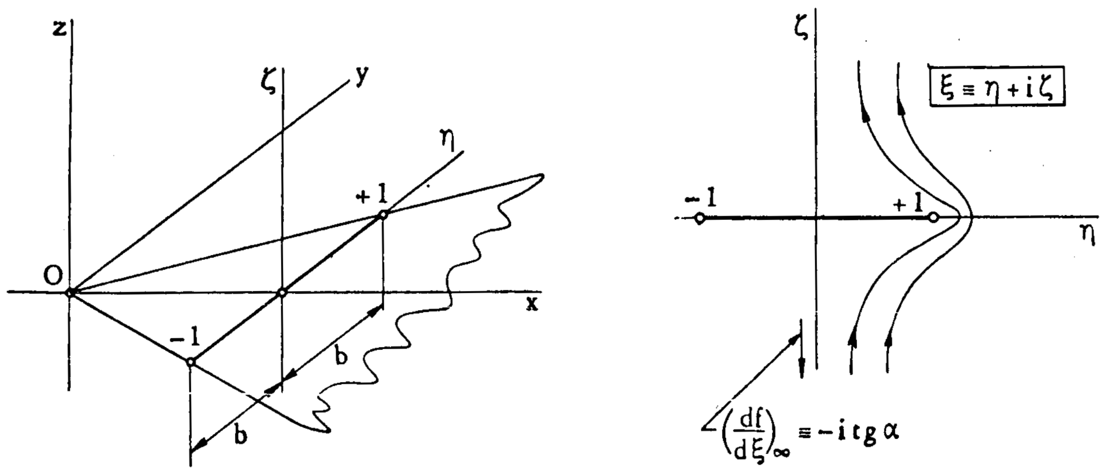

Figure 1

The components u, v, w, of the relative velocity of motion, in a non-dimensional form, are:

$$(u/V_o)\cos\alpha = 1 + \overline{\omega};\quad (v/V_o)\cos\alpha = \chi;$$
$$(w/V_o)\cos\alpha = \tau \tag{1}$$

Let us presume that the flow is conic with regard to the apex($\bar{\omega}$, χ, and τ are functions only of y/x and z/x) and that, in the transversal plane, in which the complex variables

$$\xi \equiv \eta + i\ \zeta;\ \eta = \frac{y}{x \cot \varphi},\quad \zeta = \frac{z}{x \cot \varphi},$$

the flow is solenoid; hypotheses which are not strictly compatible, as will be stated later. Then, the normalized speed ($1 + \bar{\omega}$, χ, τ) can be defined from a complex potential f (ξ) such as:

$$\chi - i\tau \equiv df/d\ \xi;\ \bar{\omega} = \cot \varphi \bullet R(f - \frac{\xi df}{d\xi})$$

The scheme of the potential continuum carries along in the transversal plane, a plane with turning at the boundaries, $\eta = \pm 1$, of the rectilinear section (-1, +1) of the axis $\bar{\eta}$ which represents the transverse section of the wing, of which the wing span, at the side x is $2b \equiv 2x \cot \varphi$.

In the half-plane section, $\eta = o$, where $\xi \equiv re^{i\theta}$, θ varies from $-\pi/2$ to $+\pi/2$, the potential in question, $f(\xi)$ and the reduced speed maybe expressed:

$$\left[\begin{aligned} f(\xi) &= -i\sqrt{\xi^2 - 1}\tan\alpha \\ \chi - i\pi &= -i\left(\frac{\xi}{\xi^2 - 1}\right)\tan\alpha \\ \bar{\omega} &= R\left(\frac{i}{\sqrt{\xi^2 - 1}}\right)\tan\alpha \cot\varphi \end{aligned}\right. \quad .$$

The bottom and top surfaces of the wing, indicated by the indices e and r, are respectively defined by $o \leq \eta_{ei} \leq 1$, $\xi_{ei} \equiv \theta_{\pm}$, and the normalized speeds take the values:

$$\bar{\omega}_{ei} = j_{ei} \frac{\tan \alpha \cot \varphi}{\sqrt{1 - \eta^2}}; \quad \tau_{ei} = o;$$

$$\chi_{ei} = - j_{ei} \frac{\eta \tan \alpha}{\sqrt{1 - \eta^2}}, \text{ with } j_{ei} \equiv \pm 1$$

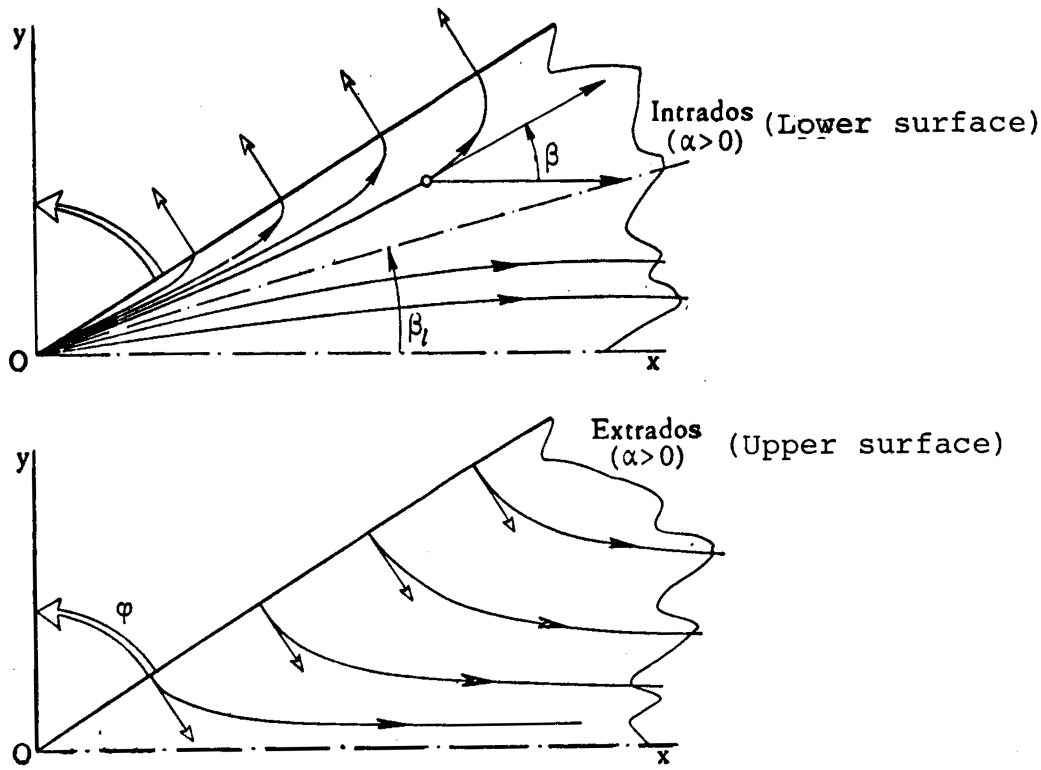

Figure 2

One deduces that the tangent to the parietal streamlines (streamlines of the flow along or attached to the surface - ed.) has an inclination, to the axis $\bar{x}$, of an angle β (see figure 2), such that:

$$\tan \beta_{ei} = \frac{\chi_{ei}}{1 + \bar{\omega}_{ei}} = \frac{- \eta}{\cot \varphi + j_{ei} \; (1 - \eta^2 \cot \alpha)}$$

<u>On the upper surface</u>, the <u>parietal streamlines</u> or <u>boundary-streamlines</u> leave (i.e., begin -ed.) orthogonally from the leading-edge and run downstream toward infinity becoming

asymptotically parallel to the $\bar{x}$ - axis.

On the lower surface, on the other hand, all of the surface streamlines begin at the apex, O. Some of these end orthogonally at the leading-edge. The others run downstream toward infinity becoming asymptotically parallel to the $\bar{x}$ - axis. These two catagories (of flow -ed.) separate along a rectilinear boundary inclined to the x - axis at an angle, β_ℓ, such that

$$\tan \beta_\ell = \cot \varphi \sqrt{1 - \left(\frac{2 \tan \alpha}{\sin 2\varphi}\right)^2}$$

This line of division of parietal flow on the lower surface exists then ($\tan \beta_\ell > 0$) only if α is small enough so that $\tan \alpha$ remains smaller than $1/2(\sin 2\varphi)$. This limit attains its maximum for the $\varphi = 45°$ case-- intermediate between large sweepback ($\varphi > 45°$) and small sweep ($\varphi < 45°$). It is understood that large angles of attack should not be considered because the theories of lift and of pressure distribution on the wings are only for rather small angles of attack.

In any case, the parietal streamlines of the lower surface at the leading-edge correspond to a turning of the adjacent fluid around the leading-edge, at right-angle to the leading-edge, and with infinite speed and therefore, with a negative infinite pressure.

In this scheme, a potential flow field in which there is a continuous flow around the leading-edge, and which, in the local plane, is perpendicular to this irrotational flow of a perfect incompressible fluid, is analagous to the present plane leaving the contour of the edge of a slender plane at an angle of attack which is not zero. Consequently, one finds again the singularities, or "physical abberations", of the rectilinear profile theory as well as the necessity of a theoretical effect of suction on the leading-edge, while, in reality, a simple detachment, or "separation," will arise along the length of the leading-edge.

Then, on the upper surface, the flow which is pouring out from the lower surface, at right angle to the leading-edge would unite in a zone of reverse motion remaining practically stationary along the length of this leading-edge. This zone would thus constitute a whirling bulb (or bulge) lengthening the leading-edge on the upper surface. It is to such a bulge that, in this case, the British denomination of "bubble" appears to correspond.

Figure 3 represents, on the whole, these characteristics of the envisaged flow.

3. APEX VORTEX AND VORTEX SHEET EN CORNET

The preceding scheme of the potential flow field was envisaged primarily because it is classic and helpful in bringing out the fundamental characteristics, and further it is very simple. However, it does not correspond to reality, except perhaps for some very small angles of incidence in which I doubt there is much interest in considering. Let us search, then for a preferable scheme of greater value.

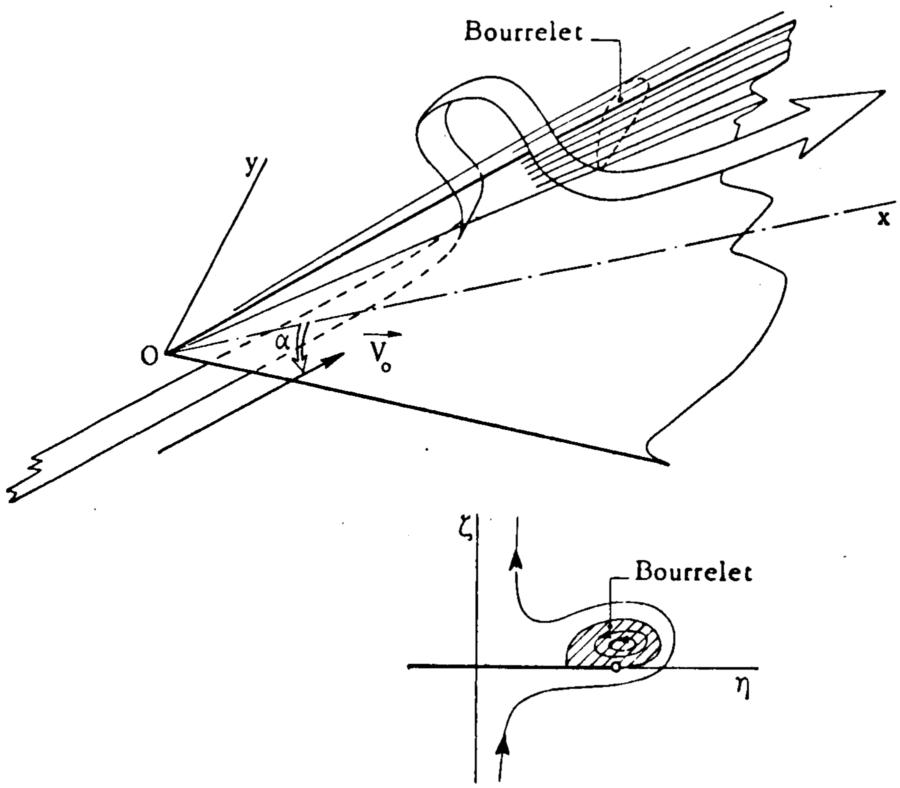

Fig. 3.

For a thin wing having delta planform, with leading-edges rather sharp, the flow around the front of the wing (with the region surrounding the apex probably being an exception) is very analogous to the relative flow in the infinite delta.

At the O. N. E. R. A. the many tests in the tunnel which have been performed in 1950-51 on wings with large sweepback and with planforms more or less like that of the delta, have made it evident that there is a formation of zones of vortices which develop above the wing and which start at the apex. These zones are rather distinctly evident as soon as the angle of attack reaches about 6 to 9 degrees for sweepback angles greater than 45 degrees.

In 1951-52, as I have mentioned above, I had a rather considerable effort developed in order to visualize, through various means, the flows in question. Apart from the particular phenomena which appear near the tips or marginal extremities of these wings as well as in certain regions of their trailing-edge, the ensemble of the vortices of the upper surface appeared, for several observers, to be summed up in two symmetric vortices coming especially from the apex and forming, above the upper surface, a "vee" lying in a plane less inclined to the free-stream velocity direction than the wing, and less open than the "vee" formed by the rectilinear leading-edges.

Observing that these vortices can only be fed and strengthened gradually by the ambient flow, I have deduced from the collection of experiments the representation of the flow in question by two <u>sheets</u> <u>en</u> <u>cornet</u> according to the sketches of figure 4.

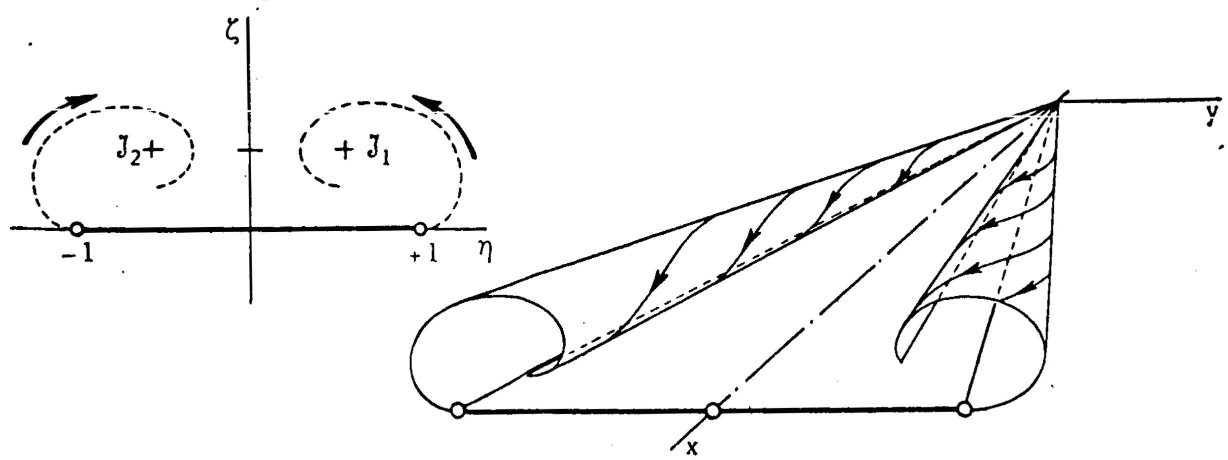

Fig. 4.

Some recent publications[1] use some schemes which appear to present a rather striking analogy to my "nappes en cornet" (sheets in cornet). In order to establish a matter of precedence, I recall that in 1952 I expounded this conception in the following terms:[2]

"the two principal apex vortices appear to me to arise, for each half-wing, rolling up en cornet from a vortex sheet being detached from the upper surface, almost orthogonally to the latter and along an almost straight line running from the apex; this line is more or less close to the leading-edge and constitutes the trace on the upper surface of a vortex "boundary-wall" between the two streams of the lower and upper surfaces."

The movement of these streams was stated as follows: The upper surface stream falling downward along from the upper side of the apex of the "tail-down arrow-shape," which constitutes the wing being studied, diverts laterally in lengthening streams downstream on the wing; equally diverted lateraly, the flow on the lower surface is caused to break (away from the surface -ed) in turning around the leading-edge toward the upper surface.

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1. One can cite, notably, a study of D. Kuchemann (report no. Aero 2540, Apr. 1955, Brit. R.A.E.).

2. Cf. M. Roy, Characteristics of the flow around a wing with extreme sweepback. (C. R. at the Academy of Science, June 23, 1952.)

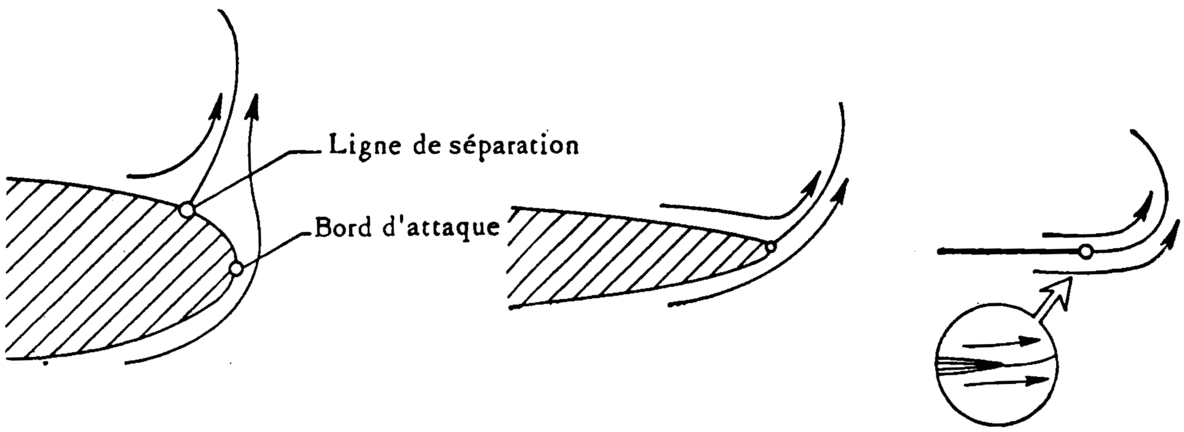
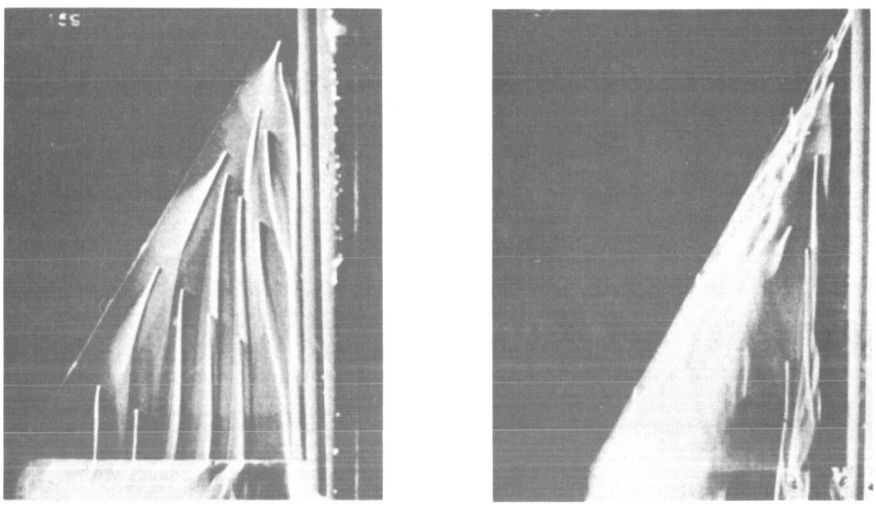


Figure 5

The difference between the line of separation cited above, or the line of departure of the vortex sheet en cornet, and the geometric line of the leading-edge is the function of the radius of curvature of the profile of this leading-edge. In particular, this difference is reduced in proportion to the reduction in this radius of curvature (figure 5), and it tends towards zero when the profile is thinned down and approaches a line, and the leading-edge becomes sharp and, preferably, tapered.

This concept was brought out before in several publications, especially in the "Recherche Aeronautique" by R. Legendre in 1953 (no. 31) and by H. Werle in 1954 (no. 41, p. 19). A concept almost identical was adopted by C. E. Brown and W. H. Michael in a very interesting paper (Jour. Aero. Sci., Oct. 1954, cf. especially page 792, fig. 1) in referring to the work of R. Legendre, but without mentioning the origin of my scheme of vortex sheet en cornet.

In order to illustrate the preceding by virtue of a simple example, figure 6 presents the visualization, by milky streaks, of the flow on the under surface (α = 20°) and on the upper surface (α = 11°) of a delta wing (ϕ = 60°), flat and thin, with a sharp leading-edge. The division of the streamlines adjacent to the lower surface and the whirling motion (vortex) of the sheet en cornet on the upper surface are particularly recognizable in these two pictures.



Lower surface (α=20°) Upper surface (α=11°)

Figure 6

4. PSEUDO-FLOW TRANSFORMATION

Let us consider the lines tangent to the components of speed situated in the transformation plane, $x = C^{te}$. They are treated continuously as streamlines of the "transformation flow". In reality, it is a question of a <u>psuedo</u> <u>transversal flow</u>, due to the fact that the flow is not solenoidal.

In fact, <u>in incompressible flow</u>, as it is evisaged here, the divergence of the speed, which is transformed is not zero, since:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{\partial u}{\partial x}$$

and $\partial u/\partial x$ cannot be identically zero in the assumed hypothesis of conicity.

Let us verify this by calculating $\frac{\partial u}{\partial x}$.

The previously defined notations will be used:

$$\left| \begin{array}{l} \frac{u}{V_o} \cos \alpha \equiv 1 + \bar{\omega} \; (\eta, \zeta) \\ \\ \frac{v}{V_o} \cos \alpha \equiv \qquad \chi \; (\eta, \zeta) \\ \\ \frac{w}{V_o} \cos \alpha \equiv \qquad \tau \; (\eta, \zeta) \end{array} \right. \qquad \left\{ \begin{array}{l} \eta = \frac{y}{\chi \cot \phi} \\ \\ \xi = \frac{z}{\chi \cot \phi} \end{array} \right.$$

One has, in the regions of irrotationality--that is to say, outside of the vortex sheets or of vortex cores--

$$(3) \quad \frac{\partial u}{\partial \chi} = \frac{V_o \cos \alpha \cot \phi}{\chi} \left[\eta^2 \frac{\partial \chi}{\partial y} + \eta\zeta \left(\frac{\partial \chi}{\partial \zeta} + \frac{\partial \chi}{\partial \eta} \right) + \zeta^2 \frac{\partial \tau}{\partial \zeta} \right]$$

an expression in which it can be seen that the second member cannot be, <u>generally</u> <u>and</u> <u>everywhere</u>, <u>zero</u>, nor even necessary negligible within approximate theories.

This second term, compared to the divergence of the transformation speed $\partial v/\partial y + \partial u/\partial z$, is of the order $\cot^2 \phi$; in this way it approaches zero as ϕ tends to $\pi/2$.

From this one can also conclude that in incompressible fluid the transversal psuedo-flow necessitates a distribution, in its plane, of sources or sinks. In a certain way of approximating, one can imagine that sources and sinks may be everywhere negligible. In fact this manner most authors have followed up to this point.

For my part, and as I have expressed in 1953 at Goettingen at the Annual Assembly, in that city, of Wissenschaftliche Gesellschaft fur Luftfahrt, I consider, on the contrary, that a more acceptable scheme than the transversal psuedo-flow must allow some distribution, at least concentrated, of sources and sinks. In my communication, at that time, which has not been published for financial reasons, I had hypothesised this distribution on the lower surface--combining two sinks with the two vortices of the lower surface and in joining compensating sources in the plane of symmetry zx of the flow.

One can envisage that sources (or sinks) which correspond to the fact that $\partial u/\partial x$ is not rigorously zero in the entire transversal plane, may be everywhere negligible. Although their effect may be summarized, even in the vicinity of the wing, as being equivalent to a certain distribution of sources (or sinks) concentrated in certain points or certain lines. In any case, if one admits the existence of a sheet, conical as a natural consequence of the admitted conic *affinity*, for the entire flow, of free vortices one will show further on that the section of such a sheet must be represented by a line of vortex sinks (or sources).

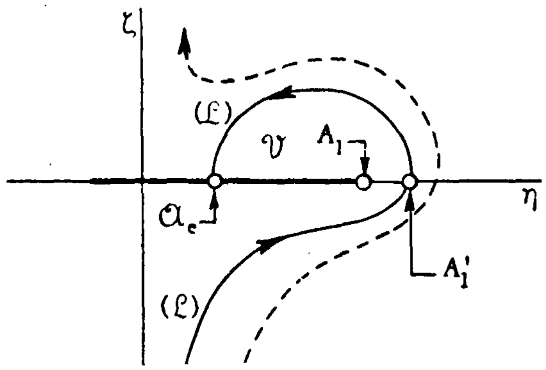


Figure 7

Here, let us remark that the *shedding* of the lower-surface flow, along the length of the leading-edge of the wing that we are considering, necessitates, *in the plane ξ of the transversal psuedo-flow*, the existence of a boundary line (ℒ) skirting, at some distance, the leading-edge and terminating on the upper surface on the positive side of the axis $\bar{\zeta}$ at a point $\mathcal{A}_e$ (figure 7).

The area, 𝒱, bounded by the axis $\bar{\eta}$ and the line (ℒ) on the side $\zeta > 0$ is supplied through a limited passage from points A_1 to A_1' on the $\bar{\eta}$ axis and thus receives a real supply (of fluid -ed.) which must be absorbed by some sinks, distributed or concentrated in the area, 𝒱. In compensation for these sinks, one must conceive in the plane, ξ, and exterior to the area, 𝒱, the existence of equivalent sources, distributed or concentrated. In fact, when one is especially concerned with evaluating, with a convenient approximation, the speeds and pressures on a wing and in the wing's vicinity, it is not prohibited, by a principal previously used, to move the compensating sources in question toward infinity to permit the use then of steady potential flow and of the normalized speed.

Moreover, as will be seen, the sheet en cornet and the conic distribution of velocity involved with it associates necessarily some weak sources or sinks with the weak vortices which constitute the sheet.

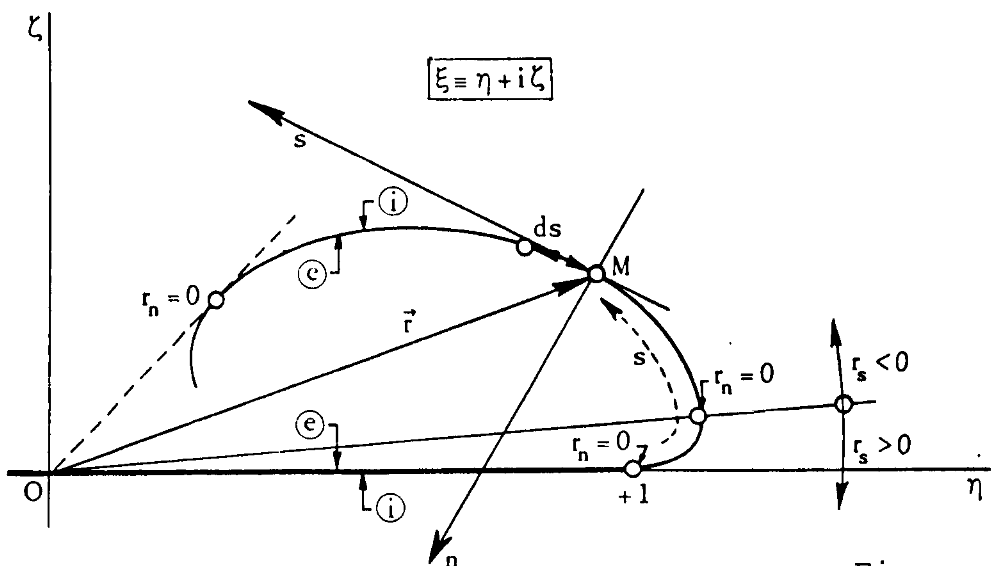


Figure 8

5. EQUATION OF THE SHEET EN CORNET

In the plane ξ (figure 8) and on the transversal section of the sheet issued from the leading-edge at the right A_1 ($\eta = +1$), let us designate a curvilinear coordinate system at the flow point (M, "downstream" from A) and name the two axis $\bar{s}$ and $\bar{n}$ which are oriented along the direction tangent and normal to the curvilinear section in question, the plane (s, n) being itself oriented in the same sense as the plane (η, ζ). Let us place the indices e and i to designate the two faces of the sheet which extend the upper (e) and lower (i) surfaces of the wing. At the flow point M on the sheet:

$$
\begin{aligned}
&G(s) = \Phi_e - \Phi_i, \\
(4) \qquad &G'(s) \equiv dG/ds = v_{se} - v_{si}, \\
&\delta(s) = v_{n_e} - v_{n_i}.
\end{aligned}
$$

V_s and V_n designate the components, along the $\bar{s}$ and $\bar{n}$ axes, of the normalized velocities (χ, τ), which is derived from the velocity potential $\Phi(\eta, \zeta)$.

According to this definition, for a positive element, ds, of the sheet:

($-G'ds$) represents the circulation directed around the element ds.

(δds) represents volume of flow of the fluid emitted albegraically by the element ds of a conic slice of the sheet of which the height, relative to the x axis, is unity.

Let us designate finally by r_s and r_n the components of the radius-vector $\overline{OM}$ of the axes $\bar{s}$ and $\bar{n}$.

An easy calculation shows that the condition of tangency of the velocity of the vortex sheet at the two faces, is expressed by the double relation:

$$(5) \qquad (v_n)_{ei} = (1 + \varpi_{ei})\, r_n \cot g\, \varphi.$$

Taking into account (2) and (4), one has:

$$\varpi_e - \varpi_i = [(\Phi_e - \Phi_i) - (\nu_{s_e} - \nu_{s_i})\, \dot{r}_s - (\nu_{n_e} - \nu_{n_i})\, r_n] \operatorname{cotg} \varphi,$$
$$= (G - G' r_s) \operatorname{cotg} \varphi - \delta r_n \operatorname{cotg} \varphi,$$

Then subtracting from (5):

$$\text{(6)} \quad \left\{ \begin{aligned} \varpi_e - \varpi_i &= (G - G' r_s)\, \frac{\operatorname{tg} \varphi}{r_n^2 + \operatorname{tg}^2 \varphi} \\ \delta = \nu_{n_e} - \nu_{n_i} &= (G - G' r_s)\, \frac{r_n}{r_n^2 + \operatorname{tg}^2 \varphi}. \end{aligned} \right.$$

According to Bernoulli's law we can state that the pressure is continuous through the sheet ($p_e = p_i$), giving the equation:

$$(1+\bar{\omega})\ (\bar{\omega}_e - \bar{\omega}_i) + \nu_s(\nu_{s_e} - \nu_{s_i}) + \nu_n(\nu_{n_e} - \nu_{n_i}) = 0,$$

This relation is finally put in the form:

$$\text{(7)} \quad \boxed{\begin{aligned} &G - G' \frac{\nu_s (r^2 + \operatorname{tg}^2 \varphi) - (\Phi + \operatorname{tg} \varphi)\, r_s}{\nu_s r_s - (\Phi + \operatorname{tg} \varphi)} = 0, \\ &\text{avec} \quad \Phi \equiv \Phi_i + G/2, \\ &\qquad\quad \nu_s \equiv \nu_{s_i} + G'/2, \\ &\qquad\quad r^2 \equiv r_s^2 + r_n^2. \end{aligned}}$$

Equation (7) of our vortex sheet <u>en cornet</u> is an integro-differential equation of a peculiar style, and it is not classic up to this time. In considering it, for example, G(s), the unknown function, enters into the expression through itself, through its derivative, and through its integrals of the second order and of the first order expressing Φ_i (s) and ν_{s_i} (s) beginning with G'(s) and r(s).

Another unknown is the "form" of the sheet, that is to say, the functions r_s(s) and r_n(s), or, if one prefers, the function ζ(n) which defines it in the plane ξ. Of course, in addition to the equation (7) expressing the continuity of the pressure through the sheet <u>en cornet</u>, one disposes of the two equations (5) expressing the tangency of the flow of the upper surface and lower surface (prolonged) to this sheet. If one considers one of these equations as defined by (4), the function $\delta(s)$, in itself considered then as known from G(s) and from $\zeta(n)$ — or $r_s(s)$ and $r_n(s)$, which follow from $\zeta(n)$, — it is only necessary to add to (7) the single existing equation (5). This suffices to determine the two unknown functions G(s) and $\zeta(n)$, on the condition that one considers as negligible the sources distributed in the plane, ξ, outside the sheet, and then that one reduces the ensemble of compensating sources of the vortex sinks of the sheet en cornet to a single source at infinity. Here the sources and sinks, the compensators, are both considered in the algebraic sense. The equation (5) can be written:

$$\mathcal{V}_{ni} \equiv (1 + \bar{\omega}_{n_i}) r_n \cot \varphi$$

$$= [1 + (\Phi_i - \mathcal{V}_{s_i} r_s - \mathcal{V}_{n_i} \cdot r_n) \cot \varphi]\, r_n \cot \varphi$$

In order to be brief, I shall not discuss here the obviously essential question of whether there exists a complete solution of the equation (7). Nor shall I discuss the fact that if such a solution is unique, the flow becomes peculiar at infinity where the compensator sources of the flow ficticiously are placed, in the plane ξ, by the assembly of the vortex sheets en cornet.

I shall only remark that, if one follows continually

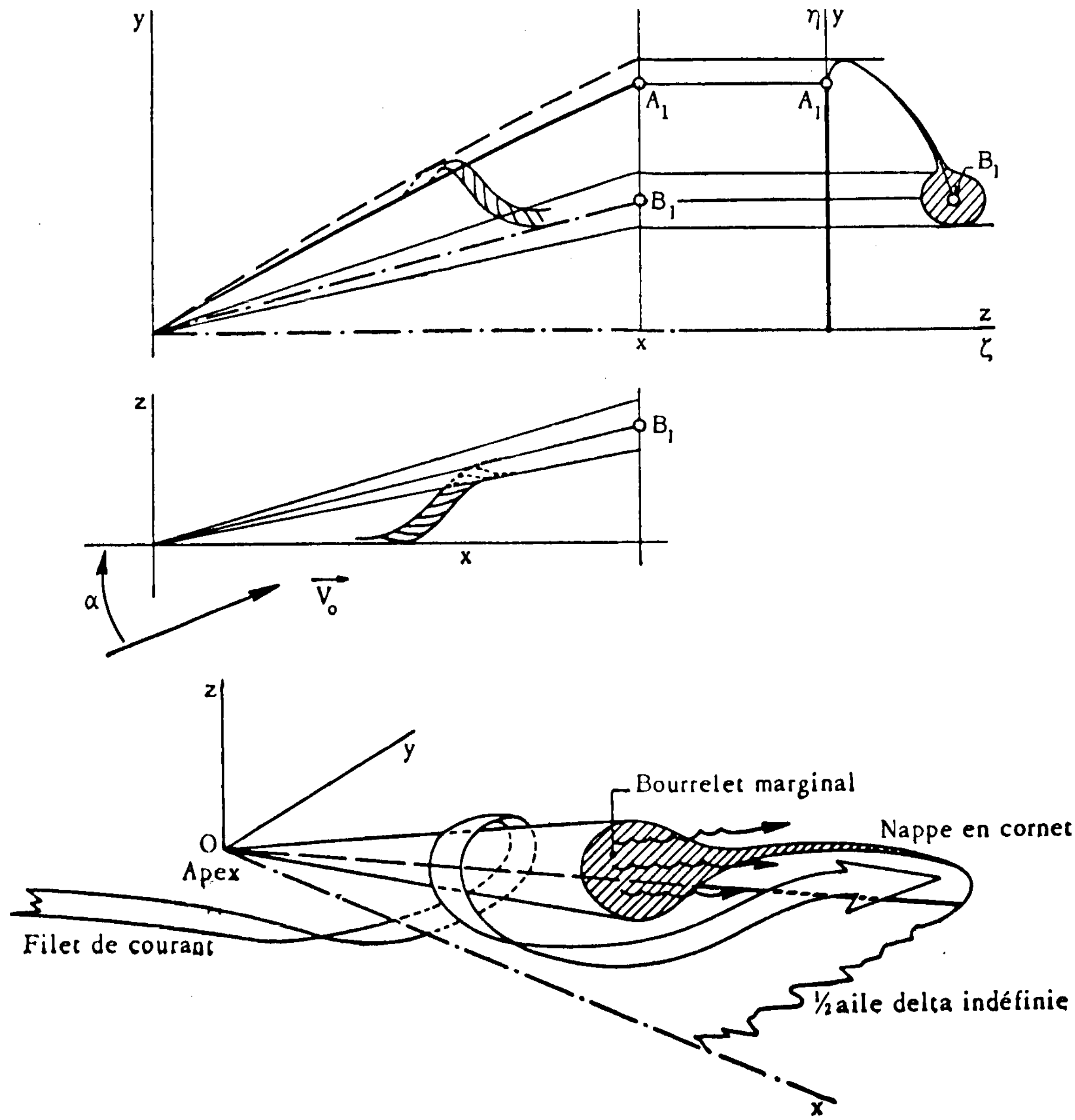

Bourrelet marginal
Nappe en cornet
Apex
Filet de courant
½aile delta indéfinie

Figure 9

the trace of the sheet, in the plane ξ, from the leading-edge, A_1, for a wing with a positive angle of attack:

$G(s)$ decreases continually, but remains positive up to the passing of the free edge of the sheet.

$G'(s)$ is always negative, and at first small.

$r_s(s)$ at first positive, quickly becomes zero, for the sheet folds back on itself in a very short space -- then becomes negative.

r_s changes sign each time that the radius-vector $\overline{OM}$ becomes normal to the trace of the sheet, that is to say, after each rotation of 180 degrees of the direction tangent $\bar{s}$ of the trace.

Therefore, at least for the initial portion of the sheet included between A_1 and the tangent furthest to the left of the origin 0 of the plane ξ (refer to figure 8), the factor $(G - G'r_s)$ the δ is certainly positive, that is, δ has the sign of r_n. This signifies that the sheet end, at least, is effectively represented, in a necessary manner, by a distribution of vortex-sinks.

If the sheet folds back more, that is, if the rolling en cornet (or "wrapping-up" -ed.) continues, the preceding character can be reversed, and it can be so each time that the sign of r_n or that of $(G - G'r_s)$ changes. Of course, the ensemble of symmetric sheets relative to the two leading-edges and the complete area $\mathcal{V}$ of figure 7 must equal a sink which absorbs the flow which crosses the line A_1 A'.

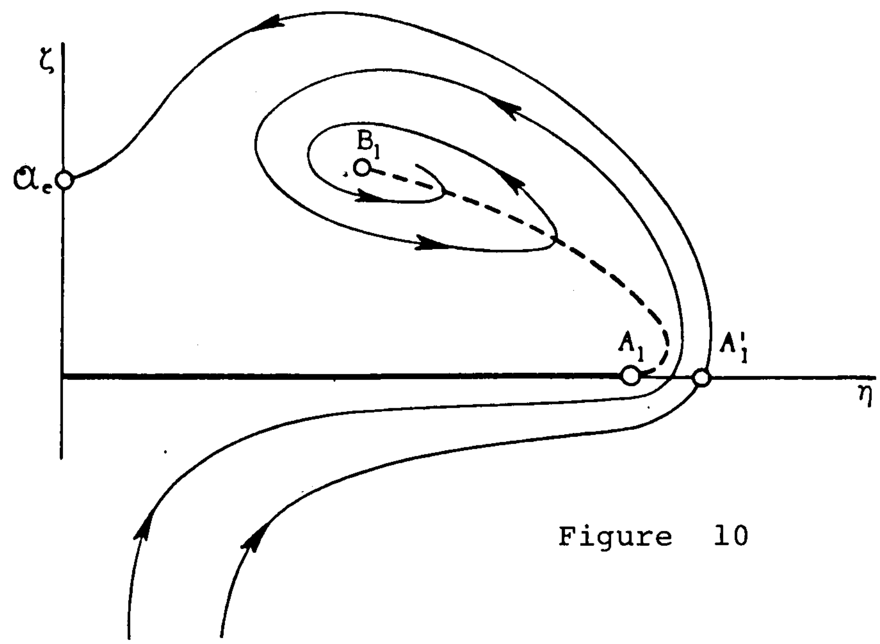


Figure 10

6. POSSIBLE FORM OF THE EDGE OF THE SHEET <u>EN CORNET</u>

The admitted conicity - a hypothesis which is well stated elsewhere - excludes that the sheet rolls indefinitely on itself, for this disposition ought to appear from the apex.

Let us assume that the rolling-up is limited, that is, that the sheet <u>en cornet</u> presents a marginal edge, indicated by point B_1 in the plane ξ(figure 9) for the sheet to the right, issued from the leading-edge A_1. Then this sheet is a conical fluid surface A_1B_1 with pressure equal on both faces at each point. This surface extends the wing surface and the <u>marginal edge B_1 of this sheet is, in a</u>

manner, substituted for the leading-edge A_1 of the plane wing—extra thin and with sharp-edges.

This sheet is thus formed to avoid the direct turning of the fluid around the sharp leading-edge at A_1, which is not possible by continuous potential flow. One must admit, on the contrary, that the flow turns around the marginal edge B_1 of this sheet. One will note, in passing, that if it did not thus procede G(s) and G' = dG/ds would be zero at B, or one would have the cancellation of strength of the circulation, and at the same time, of the surface outflow of the sheet *en cornet*.

The turning about B_1 by the transverse psuedo-flow is, beside being made evident by the present experiment, according to the words of Pascal, "the master that we must follow."

However, it is not possible to neglect the viscosity of (the fluid at -ed.) point B_1 nor in the vicinity of B_1, for it alone prevents the speed from increasing indefinitely as it would in the case of a perfect fluid turning the sharp edge B_1 of our schematic sheet.

Because of the viscosity, there is formed at B_1 a conical "bubble" (actually a rotational core -ed.) fed progressively and continually by the vortex filaments which constitute the sheet and which follow, on the sheet, a line from the leading-edge at A_1 so that they finally, and rather rapidly, become oriented along the axis of the marginal bulge when they reach the latter (refer to figure 9).

In the case, depending on the angle of attack α, where the angle of turning of the sheet — the total angle of rotation in the plane ξ of the directional tangent $\bar{s}$ along the trace of the sheet — is less than π, the velocity vectors of the transverse pseudo-flow are presented as the schematic of figure 10. In this figure, the terminal point $\mathcal{Q}_e$ of figure 7 has been placed hypothetically on the positive part of the axis $\bar{\zeta}$.

Let us stress this concept of the "bourrelet marginal de la nappe en cornet" — padded edge of the rolled-up vortex sheet.

It is a rotational region. The viscosity of the real fluid, which cannot be neglected in this zone with its extreme velocity gradients, causes the entrainment by this bulge (or vortex core -ed.) of the fluid field which turns around it.

This entrainment occurs continually in proportion to the distance from the apex, continually enlarging the rotational core, the conic form of which blends with that of the (vortex) sheet en cornet, which agrees with our fundamental hypothesis of conicity, originating at the apex, of the entire flow.

The formation of the edge bulge can be represented initially by neglecting the viscosity as shown in figure 11.

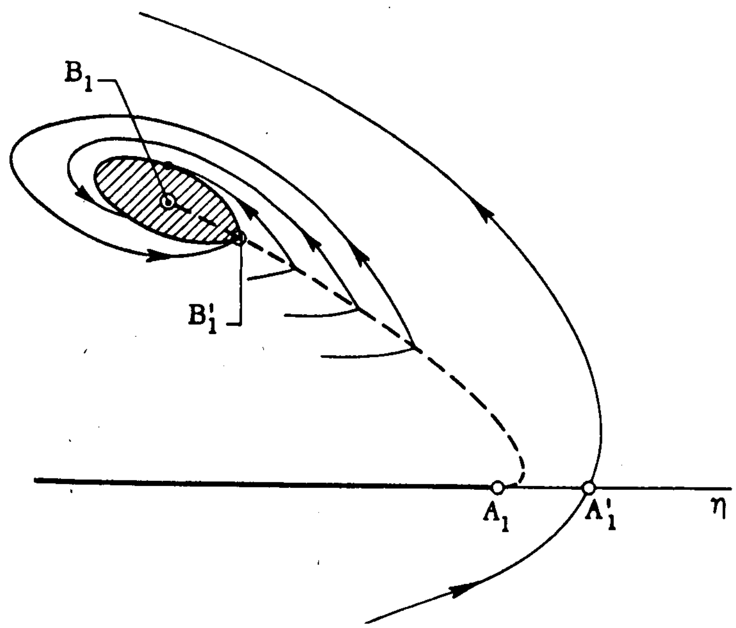


Figure 11

The vortex core (shaded) absorbs at its edges the flow of the transversal pseudo-current which has crossed the boundary A_1 A' and which has not been absorbed through the trace A_1 B_1' (instead of A_1 B_1) of the sheet en cornet. One can imagine that this core may be in constant rotation, according to a conception recently developed by R. Legendre for a cylindrical pad (bubble) on the leading-edge of a rectangular wing plane. Let us emphasize, at any rate, that in proportion to the distance from the apex, the circulation of the marginal pad and its contained flow volume increase proportionally to the abscissa x; first, by the continual contribution of the vortex filaments from the sheet en cornet (of reduced section issued from A_1, but limited to B_1') and second, by the absorption of inflow through the lateral surface of the bubble (according to the effect of sinks on the contour of the core in the plane ξ).

The presence of this bubble, its progressive development, and its stability linked to that of the vortices within a fluid having negligible viscosity (outside these vortices), are very well corroborated by experiment. Especially must there be explained the double streaks, with multiple turns, observed on the streamlines adjacent to the padded edge of the rolled-up vortex sheet.

It is, moreover, in the same way explained, the formation of the _tip vortices of the straight wing or ones having very small sweepback_ and having an almost elliptical lift distribution. At the tips of these wings, and from the leading-edge itself, if the latter curves rapidly downstream in the region of the tip, there forms a (vortex) sheet _en cornet_ which creates its own circular core as it is in this one, very concentrated and very durable. This core, or bubble, constitutes the _tip vortex_ of the wing in the usual sense.

7. PRACTICAL SCHEME OF FLOW

The delicate question remains to define a scheme being inspired by the preceding — which comes directly from experiment — in such a manner that this scheme may be practical, that is, that it furnishes a good enough approximation of the distributions of speeds and pressures on the wing and around the wing and that it simplifies the calculations sufficently.

I have insisted upon the condition "rational" to which my concept of the sheet _en cornet_ is subjected in order to respect the laws of fluid mechanics, taking into account an intervention of the viscosity; so as to explain the detachment of this sheet from the leading-edge if it is sharp, or from an adjacent line if it is more or less rounded, and thus to explain the formation of a _rotational core on the edge of the sheet_.

Among these rational conditions appears, especially, the equilibrium of pressure, at any point on the sheet, between the two faces; and a finite difference between the direction of the flow velocity from the upper surface and that of the flow from the lower surface.

I have emphasized also the theoretically necessary connection of weak sink (or sources) of weak vortices of the sheet _en cornet_, in the assumed hypothesis of conicity as well as the approximate character of the concomitant hypothesis of solenoid flow for the transversal psuedo-flow outside of the preceding singularities.

Recently, I have made, with the collaboration of Mr. P. Duban, several attempts at maximum simplification of the scheme of flow. All of these attempts in which the sheet _en cornet_ was represented only fragmentarily have furnished some results which, while very acceptable in several interesting respects, entail some local unpleasant faults.

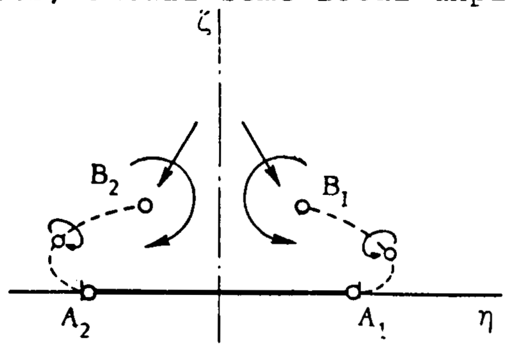


Figure 12

Another attempt, at present in the course of calculation, involves a sheet <u>en cornet</u>, with a continuous distribution of vorticity from A, to B_1 and completed at B_1 by a concentrated vorticity-sink (figure 12). In spite of this stylization taken from the true structure of the sheet <u>en cornet</u>, the calculations are far more difficult than the preceding. The results will be presented later if there will be a place for them.

In any case the representation (at B_1 and B_2) of the marginal vortex bubble by a concentrated vortex sink, that I proposed at Goettingen in 1953, appears to me to gain credence, by what the visualization of the real flow teaches, and at the same time, by a reasonable concern for simplification of calculations. The calculations are concerned with the question of evaluating speeds and pressures on a wing or in its immediate vicinity.

8. THE TEARING OF THE SHEET *EN CORNET*

It is evident that, in reality, and for a delta wing of great depth (long chord -ed.), the conic flow is only approximate, and only for a limited portion of the wing. The viscosity exercises a cumulative influence, with the length of the leading-edge and distance from the apex, on what causes the formation and detachment of the sheets *en cornet*. This influence progressively causes the real flow to diverge from the idealized flow, that one assumes elsewhere for a perfect fluid.

One can, from this, imagine that at a certain distance from the apex, the sheet *en cornet*, formed from this point, breaks away from the leading-edge and that a new piece of sheet *en cornet* is formed from this point of tearing. This new sheet, in turn, will terminate at a point more downstream on the leading-edge, and so on.

Thus, several "bubbles", sorts of concentrated vortices, can appear above and along a delta wing, vortices which then roll more or less rapidly, on each other.

The effect of interaction of the trailing-edge on the flow around the leading-edge can equally favor the tearing of a sheet *en cornet* before its development attains the delta wing tip.

Some number of observations of flow visualized at the O. N. E. R. A., on thin wings with strong sweepback, delta wings, as well as some streamline bodies, appear to me as basis for the preceding statement. I limit myself here to emphasizing the influence, evidently major in this instance, of the Reynold's Number relative to the magnitude of the obstructions to the meaning of the flow field.

Translation of: SUR L'ECLATEMENT DES TOURBILLONS D'APEX D'UNE AILE DELTA AUX FAIBLES VITESSES

From La Recherche Aeronautique, No. 74, Janvier-Fevrier, 1960.

ON THE BURSTING OF THE APEX-VORTICES OF A DELTA WING AT LOW SPEEDS

by

H. Werle, Research Engineer of the O.N.E.R.A.

SUMMARY

In a conference held recently in Hanover, Germany (1), Mr. Maurice Roy, director of the O. N. E. R. A., again discussed and developed his concept of the flow around a delta wing, operating at an angle of attack, characterized by the rolling-up into a "cornet" of the vortex sheet of the upper surface of the wing.

In the course of this report, some visualizations (of the flow), obtained in a hydrodynamic tunnel (2), illustrated an analysis of the entire flow as well as certain local details of the various types of flows studied.

The experimental results presented below enter the framework of the general study. They concern a detail, as yet little understood, of the vortices from the apex: the phenomenon of the bursting of the vortices under the action of turbulence. The studies carried out in a hydrodynamic

tunnel have permitted an exact determination of the mechanism of the explosion, the influence of the downstream conditions, and of the principle parameters.

I. MECHANISM OF THE BURSTING

Let us first recall briefly the structure of the (vortex) sheet <u>en cornet</u> (3) and (4):

The marginal bulge or bubble of the sheet - a kind of a conical shaped core rolling up into one piece - is maintained all along the leading-edge. All colored streaks emitted at the leading-edge twine around the axis describing a helix of constant diameter and of slightly increasing rate. The diameter of the helix is an increasing function and is approximately linearly proportional to the distance from the apex to the point of emission on the leading-edge. In particular, an emission near the apex makes the axis of the bulge easily seen: see figure (1). This bulge is completely laminar near the apex.

The appearance of turbulence in the downstream part of the bubble disorganizes the mechanism of the flow by provoking a veritable explosion of the core:

The colored stream emitted near the apex and describing a helix of a very reduced diameter around the axis of the laminar part of the core drastically transforms itself through transition (E) into a helix of increasing diameter while starting to comprise a pocket of turbulent fluid: see figures 1 to 3.

II. INFLUENCE OF DOWNSTREAM CONDITIONS

Figures 4 to 7 show that the phenomenon is very sensitive to the downstream conditions of flow:

— A suction at the interior of the turbulent pocket permits the retardation or even the stoppage of the bursting (4).

— A solid or fluid obstacle placed in the same conditions brakes the flow and provokes an inverse effect. (fig. 5 & 6).

— A jet in the direction of the flow, sent, for example, from the lower surface side, accelerates the fluid which has as a consequence the recession of the explosion at the same time as the curving of the axis of the vortex toward the jet. (fig. 6)

All these test prove that the phenomenon of bursting is tied to the very rapid deceleration of the fluid, confirmed by certain hypothesis set up by Bryer and Lambourne (5).

III. INFLUENCE OF THE PARAMETERS

A comparison of figure 8 (view at the right) and figure 1 shows that with the incidence (angle of attack) $\underline{i}$ and the sideslip (angle of yaw) $\underline{j}$ constant, the explosion

E approaches the apex whenever the Reynolds number R_ℓ (based on velocity V_o) increases; E tends toward a limit which is the position actually reached for $R_\ell = 10^4$.

An increase of incidence, i, accomplished with j and R_ℓ constant, is marked by a progressive movement (from the downstream beyond the trailing edge) from point E toward the apex, thus reducing, in a continuous manner, the laminar part of the core (figure 8).

Given the structure of the wing (constant thickness = 1 mm.), the sideslip angle, j, of the wing can be used to make an apparent modification of the sweepback of the leading-edge. This sweepback becomes 60° + j or 60° - j depending on the edge being considered (the right half-wing or the left half-wing).

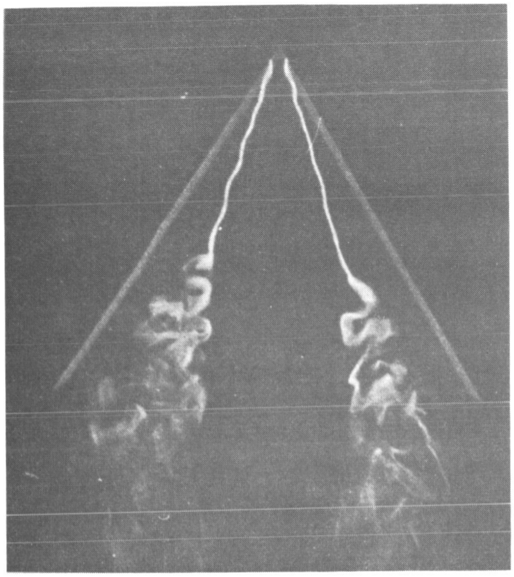
The influence of the sideslip (or yaw), with i and R_ℓ constant, appears clearly in figure 9 (in comparison with figure 8):

On the half-wing for which the sweepback effectively increases, one observes the recession of the bursting toward the trailing-edge, whereas, on the contrary, the turbulent pocket continually moves forward toward the apex as φ_{EFF} decreases.

The thickness of the wing and the radius of its leading-edge have an effect on the explosion E analogous to that of the incidence; situated in identical test configuration (i, j, V_o, R_ℓ, φ_{BA}), the occurrence of the bursting is retarded when the wing considered is thicker and the radius of the leading-edge is greater: (observe and compare figure 10 with figure 1).

Finally, the planform of the wing has a not negligible effect on the appearance of the phenomenon (see figure 11, and compare with figure 8 (b). In effect, in the case of a slightly-opened cone, resembling a delta wing, with very large sweepback, the vortices issuing from the point do not burst in the immediate proximity of the model, at least in the usual instance. It is the same for the tip vortices of a rectangular wing of short span or of an elliptical wing.

Manuscript submitted Dec. 29, 1959



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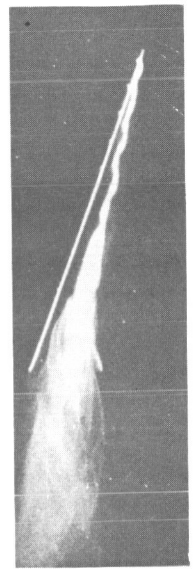

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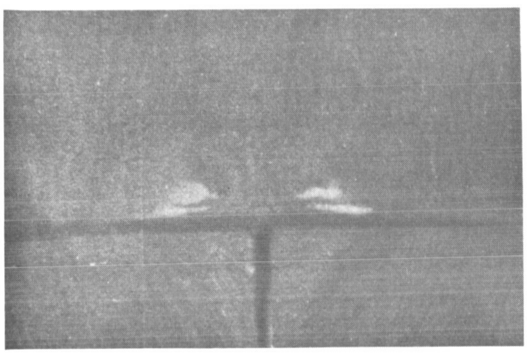
Fig. 1 - Mechanism of the bursting:
(a) View of the upper surface. (b) View in perspective.
(c) View of the profile.



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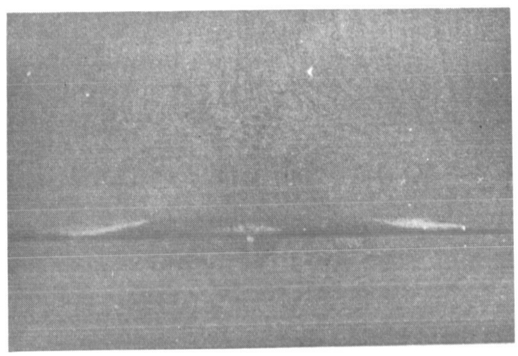

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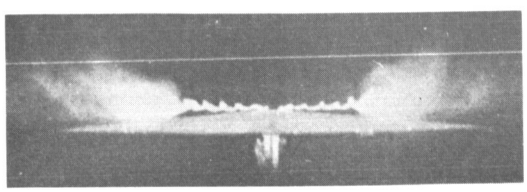

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Fig. 2 - Mechanism of the bursting:
(a) Transversal section of the explosion up-stream from the explosion.
(b) Transversal section of the explosion downstream from it.
(c) Downstream view of the phenomenon.

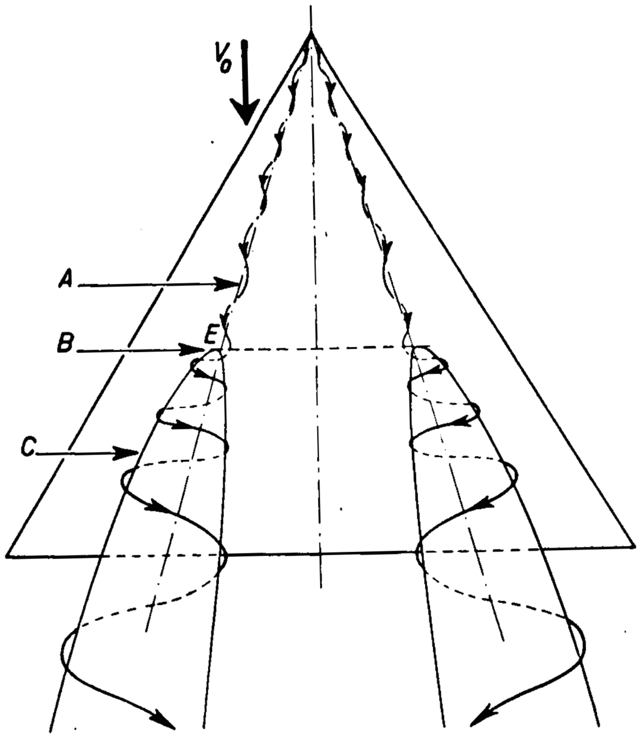


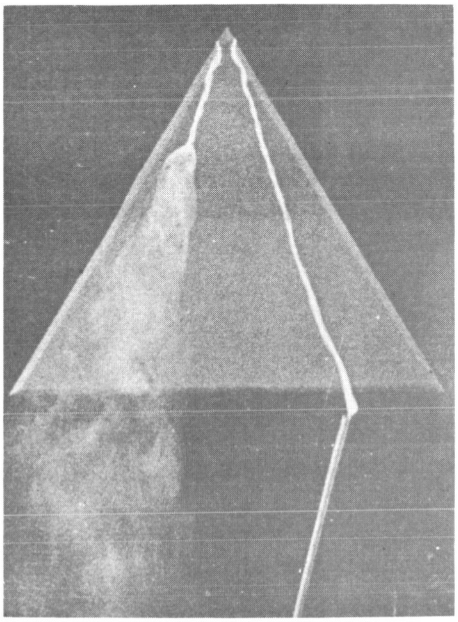
Fig. 3 - Mechanism of the bursting.

Schematic diagram of the mechanism of the bursting. Thin delta wing with a sharp leading-edge; incidence, i=20°; sideslip, j=0°; $R_\ell \simeq 5\times10^3$; V_0= 5 cm./sec.; sweepback of the leading-edge, ϕ_{BA} = 60°; chord at the wing root, ℓ=100 mm.; e/ℓ = 1%.

(A) Axis of the apex votex.

(B) Point of bursting of the vortex.

(C) Pocket of turbulent fluid.



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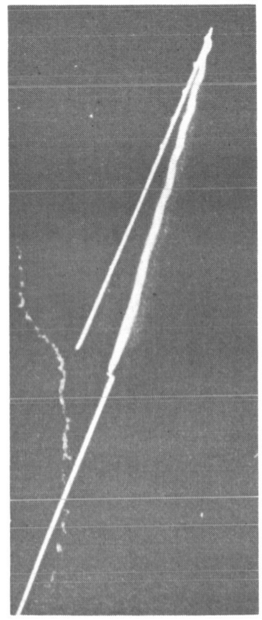


b

Fig. 4. Influence of downstream conditions: Thin delta wing with sharp leading-edge.
$i=20°$; $j=0°$; $V_0=5$ cm./sec.; $R_\ell = 5x10^3$.
(a & b) Influence of suction.

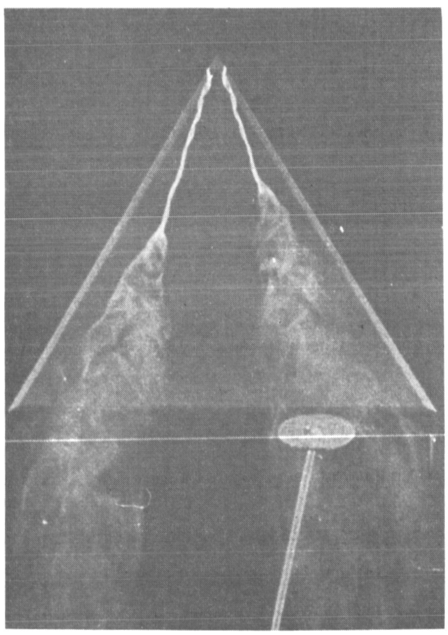
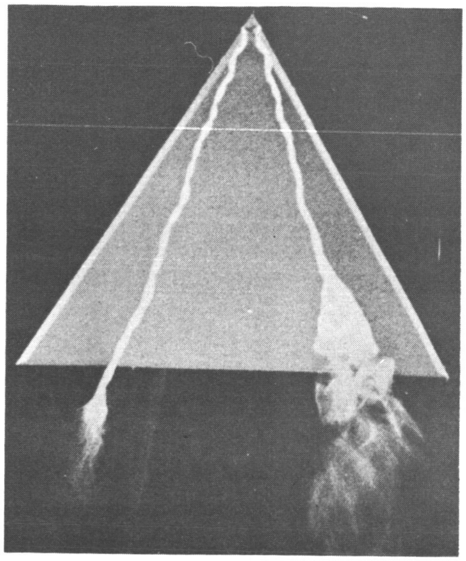


Fig. 5. Influence of downstream conditions: Thin delta wing with sharp leading-edge.
$i = 20°$; $j = 0°$; $V_0 = 5$ cm./sec.; $R_\ell = 5x10^3$.
Influence of an obstacle.



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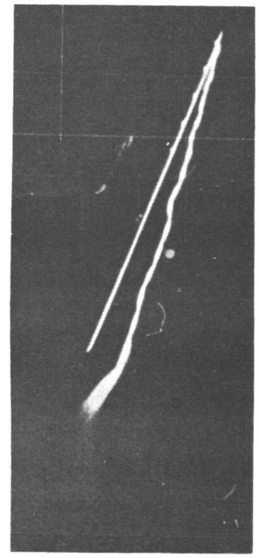

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Fig. 6. Influence of downstream conditions: Thin delta wing with sharp leading-edge.
$i=20°$; $j=0°$; $V_0 = 5$ cm./sec.; $R_\ell = 5x10^3$.
(a & b) Influence of a jet in the direction of flow.

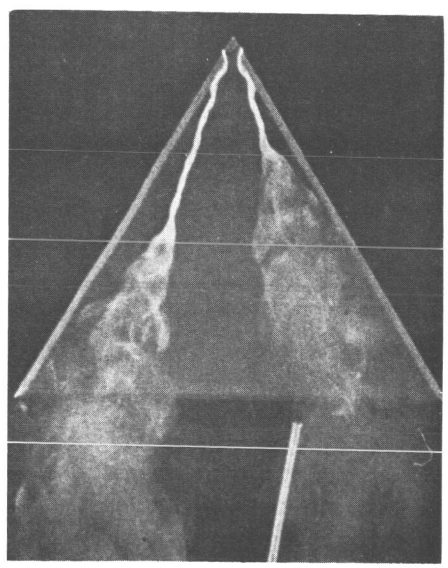


Fig. 7. Influence of downstream conditions: Thin delta wing with sharp leading-edge.
$i = 20°$; $j = 0°$; $V_0 = 5$ cm./sec.; $R_\ell = 5x10^3$.
Influence of a jet against the flow.

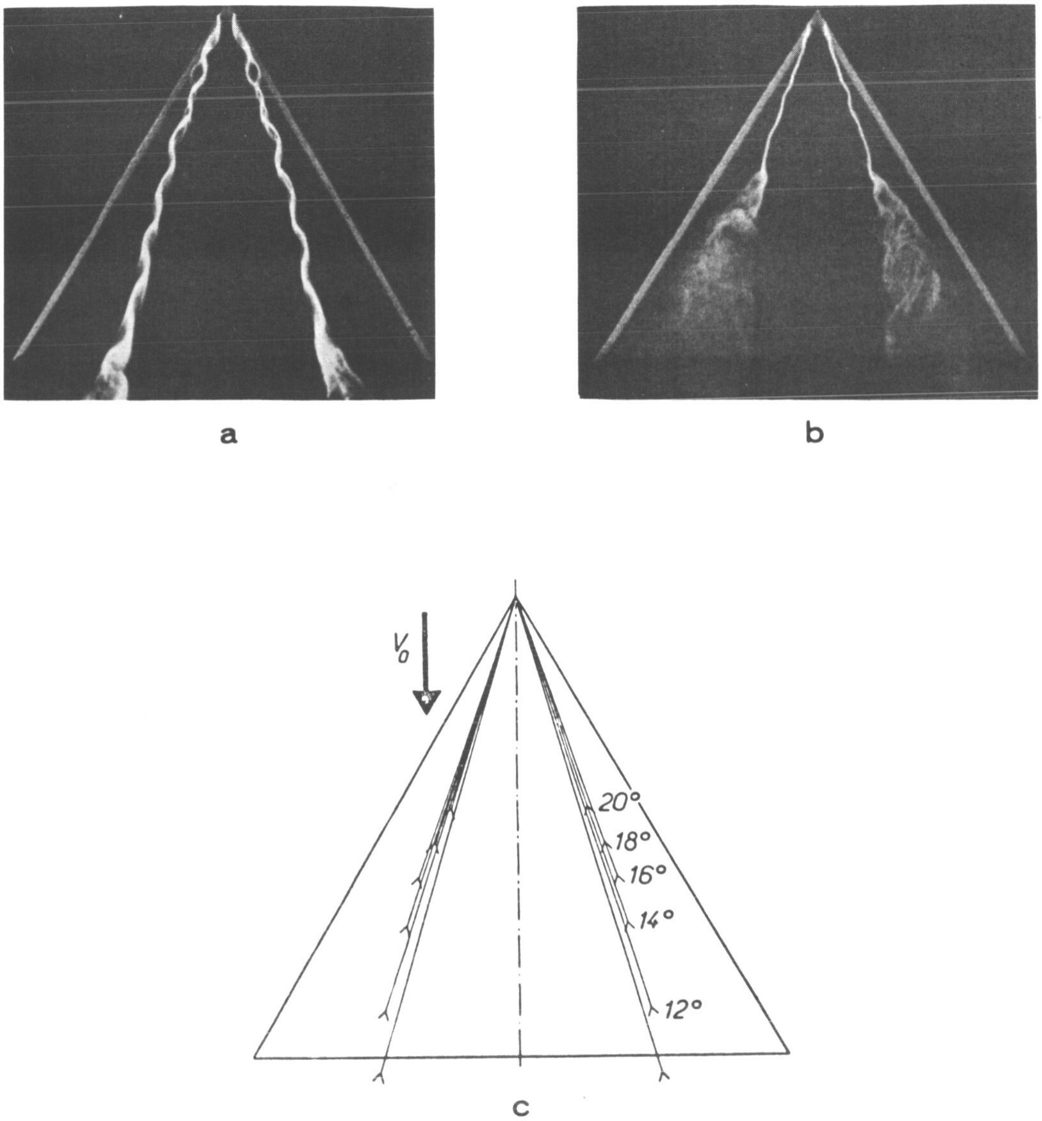


Fig. 8. Influence of incidence angle, i.
(a) i = 12°, j = 0°. (b) i = 20°, j = 0°.
(c) Schematic gives the displacement of E as a function of i. Tests carried out with j = 0°; V_0= 10 cm./sec.; $R_\ell = 10^4$; chord at the root of the wing, ℓ = 100 mm.
The figures shown on the schematic give the value of the incidence, i.

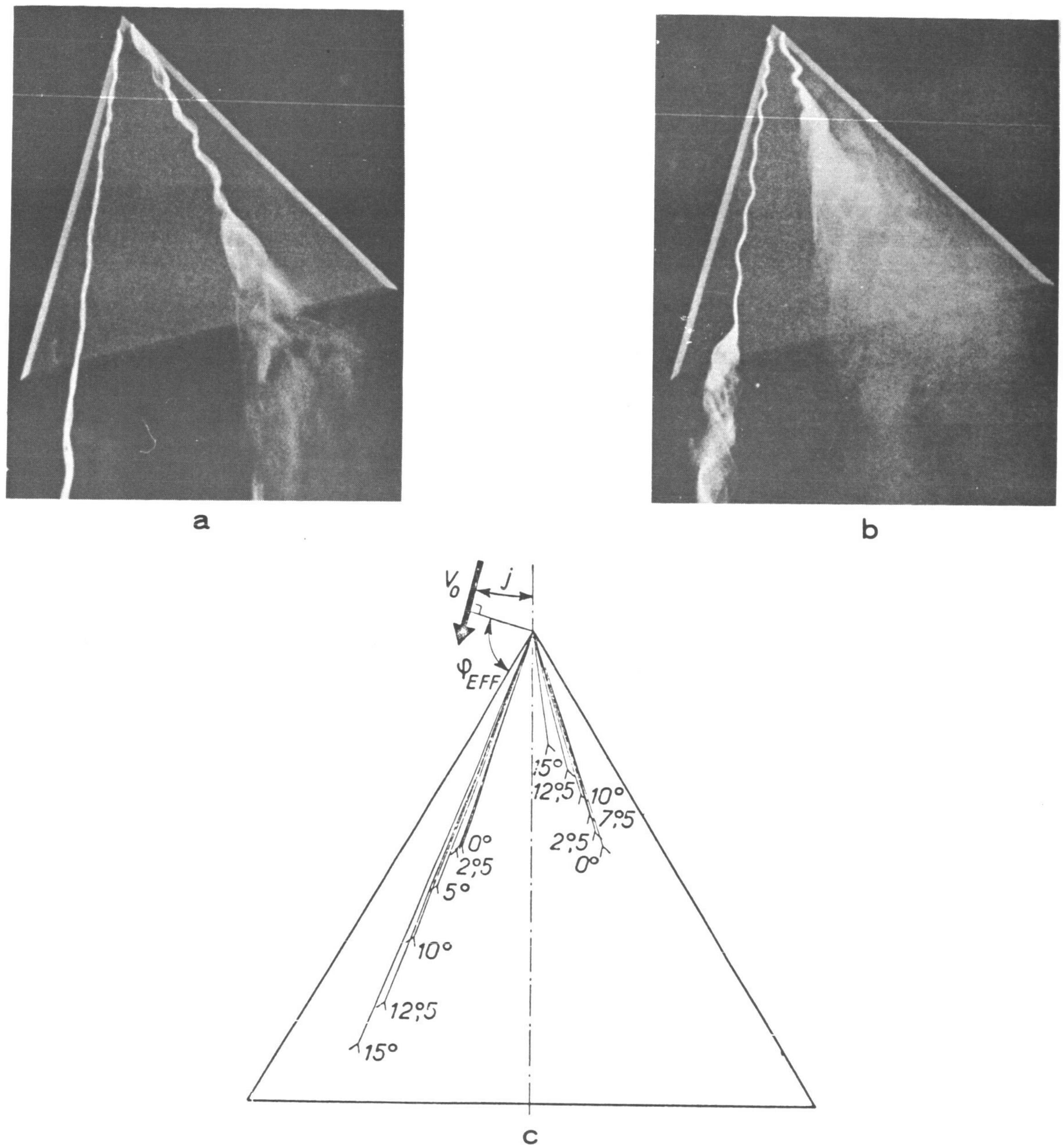
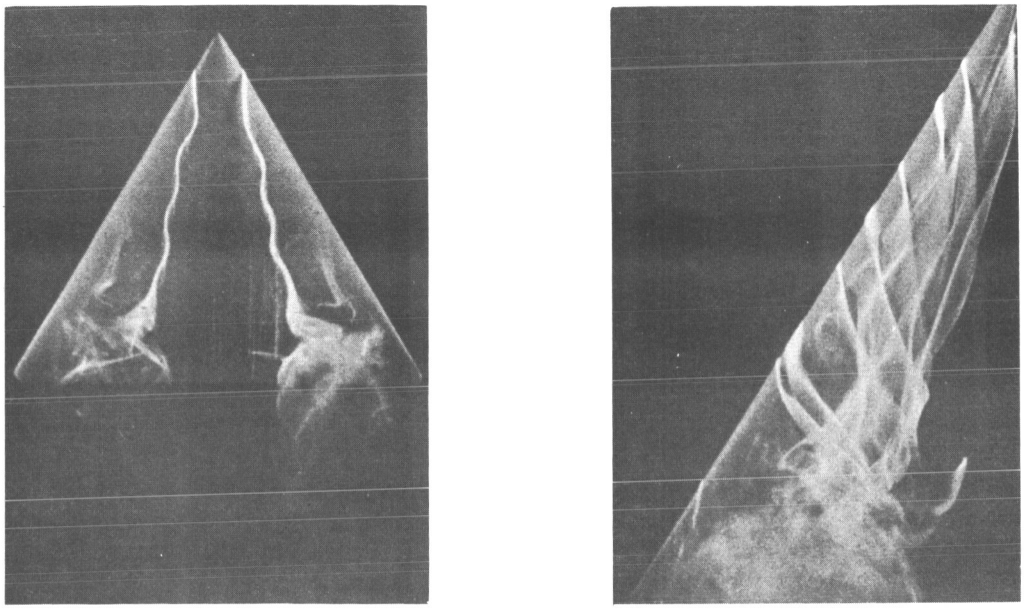


Fig. 9. Influence of the side-slip.

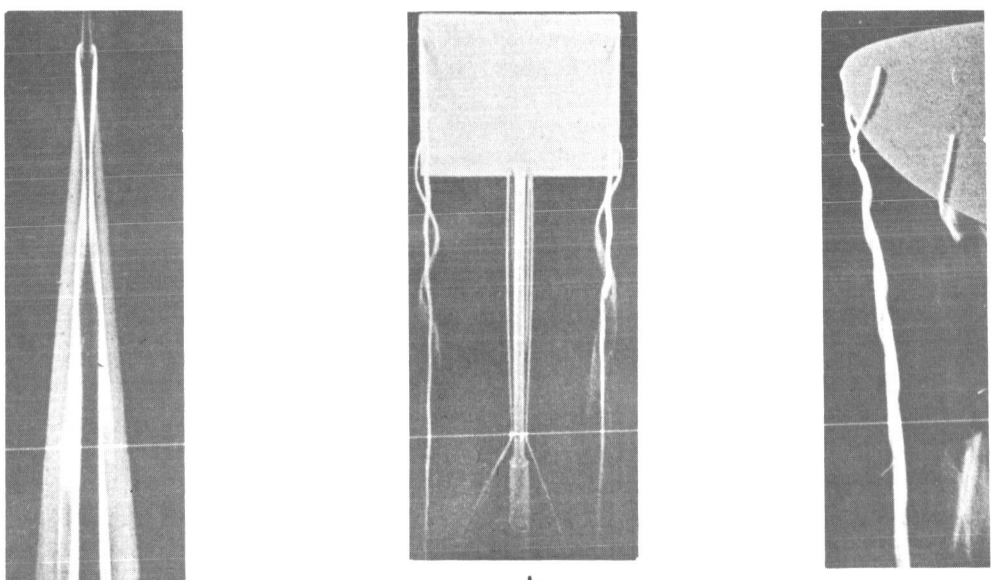
a) $i = 12°$, $j = 15°$. b) $i = 20°$, $j = 15°$.

c) Schematic gives the displacement of E as a function of j. Tests carried out with $i = 20°$; $R_\ell = 10^4$; chord at the root of the wing, $\ell = 100$ mm.; $V_0 = 10$ cm./sec.. The figures on the chart give the value of the side-slip angle.



a b

Fig. 10 - Influence of the thickness of the wing.
a) e/ℓ = 10% (median plane); $i=20°$; V_0 = 5 cm/sec.
b) e/ℓ = 16% (median plane); i = 25°; V_0 = 5 cm/sec.



a b c

Fig. 11. Influence of the form of the model.
a) cone; view of the top surface; i = 12°, V_0= 10 cm./sec.
b) rectangular wing; view of top; i = 12°, V_0= 10 cm./sec.
c) elliptical wing; lower surface tip; i=11°, V_0=10 cm/sec.

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